Cosmological Applications of the Frieden-Soffer Nonextensive Information Transfer Game

L. P. Chimento[a, c] F. Pennini[b] , and A. Plastino[b, c1, 2, 3]

[1a] *Departamento de Física, Facultad de Ciencias Exactas y Naturales,*
Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina.
[2b] *Physics Department, National University La Plata, C. C. 727,*
1900 La PLata, Argentina.
[3c] *Argentine National Research Council (CONICET).*

We show how the demon of Frieden and Soffer, working in a non-extensive statistical scenario, is able to devise solutions to some of Einstein's field equations by recourse to nonlocal changes of variables between appropriate differential equations. It is seen that a variety of cosmological problems involving Einstein's field equations can be reinterpreted as situations in which the pertinent solution is obtained, with tools of Statistical Mechanics, *in a nonextensive Tsallis scenario.*

PACS numbers: 05.20.-y, 89.70.+c, 98.80.Hw, 95.30.Sf.
Keywords: Fisher Information, Tsallis Thermostatistics, Frieden-Soffer Game, Cosmological Models.

arXiv:1207.3377v1 [math-ph] 14 Jul 2012

I. INTRODUCTION

Lie's Group theory approach to the study of differential equations has had a profound and enduring impact in Theoretical Physics, in particular in what respects to the invariance of differential equations under point transformations [1]. He showed that the one-dimensional free particle equation exhibits the symmetries corresponding to the eight-dimensional SL(3,R) group of point transformations and that this is the maximum number of symmetry generators for a second-order differential equation of the form [1]

$$\ddot{y} + h(\dot{y}, y, x) = 0. \tag{1}$$

Of course, one often encounters situations in which one deals with a smaller number of generators. In a variety of such cases [2], equation (1) can nonetheless be recast in the form of a free particle equation by recourse to a *non-local variable transformation.* Now, as nonlocal transformations can change both the number of symmetry generators and the very physics of the original problem, they become a powerful tool for theoretic analysis. Einstein's field equations, whose nonlinear nature can indeed be faced with Lie's weaponry, constitute a particularly relevant example in this respect.

It is via this equivalence (under nonlocal variable transformations) between different physical problem that we will search here for connections between

- Cosmological problems in General Relativity, on the one hand, and
- Information Theory Concepts pertaining to *both* Fisher's Information measure for translation families (FIM) [3–14] and Tsallis' Nonextensive Thermostatistics (NET) [15–24], on the other one.

This search is motivated by the recent findings of Ref. [25]: an intriguing relation between homogeneous and isotropic spatially flat cosmological models with no cosmological constant, on the one hand, and FIM-probability distributions that solve *the diffusion equation, as adapted a non-extensive ($q = -1$) Tsallis environment*, on the other one.

FIM's applications to diverse problems in theoretical physics have been pioneered by Frieden, Soffer, Nikolov, and others [5–14], who have unveiled many FIM properties and clarified its relation to Shannon's logarithmic information measure

$$S = - < ln f >, \tag{2}$$

where f is, of course, a probability density and $< A >$ stands for the trace of (fA).

General Relativity is a classical theory of gravitation that has been successfully applied for solving problems concerning astronomical scales of length and time, such as the formulation of cosmological models. Einstein's equation for the classical geometry adopts the appearance

$$R_{ik} - \frac{1}{2} g_{ik} R = 8\pi G T_{ik}, \tag{3}$$

where R_{ik} is the Ricci tensor, R the scalar curvature and T_{ik} the energy-momentum tensor of matter and fields, with $i, k = 1, .., D$, and D, the spacetime dimension.

During the last few years much attention has been paid to models in which the universe, at very early stages of its evolution, was expanding [26–29]. The well known Big Bang model suggests a spatially homogeneous and isotropic space-time, stemming from a singularity in the remote past [30].

Here we will show that a generalization of the so-called Frieden-Soffer principle of Extreme Physical Information [11], within the framework of a non extensive setting, may yield original insights into the workings of some of Einstein's field equations.

As a first step in such a direction, and in order to establish a convenient notation, we begin by refreshing, in Section II: i) the salient points of Fishers's measure tenets for translation families, ii) their extension to non-extensive settings, and iii) the Frieden Soffer Principle concerning extremalization of Physical Information [11]. In Section III we discuss the application of this Principle to a non-extensive environment. We present the main (cosmological) results of the present Communication in Section IV and, finally, some conclusions are drawn in Section V.

II. A REVIEW OF FUNDAMENTAL NOTIONS

A. The estimation theory bound

In this work we shall concern ourselves *exclusively* with Fisher information measures I for translation families, that are invariant under Galilean and Lorentz translations [31]. They refer to a measure of the inverse uncertainty in determining a *position* parameter by a maximum likelihood estimation [32].

Estimation theory [5] provides one with a powerful result that constitutes our stepping stone. Consider an isolated many-particle system that is specified by a *position* parameter x and let $f(x)$ describe the probability density (pd) for this parameter. The mean value of x for this pd is η. Suppose the pd is unknown, and one wishes to determine it. Estimation theory asserts [5] that the best possible estimator η_{est} of our parameter, after a very large number of samples is examined, suffers a mean-square error e^2 from η that obeys a relationship involving Fisher's I, namely,

$$e^2 = \frac{1}{I}, \tag{4}$$

where Fisher's information measure I (for tranlation families) reads

$$I \,=\, \int dx\, f(x)\, \{\frac{\frac{df}{dx}}{f(x)}\}^2, \tag{5}$$

i.e.,

$$I \,=<\, \{\frac{\frac{df}{dx}}{f(x)}\}^2\, > . \tag{6}$$

Any other estimator must have a larger mean-square error. The only proviso to the above result is that all estimators η_{est} be unbiased, i.e., satisfy

$$< \eta_{est} >=\, \eta. \tag{7}$$

Thus, Fisher's information measure for translation families has a lower bound, in the sense that, no matter what parameter of the system we choose to measure, I has to be larger or equal than the inverse of the mean-square error associated with the concomitant measurement. This result, i.e.,

$$I\, \geq\, \frac{1}{e^2}, \tag{8}$$

is referred to as the Cramer-Rao bound, and constitutes a very powerful statistical result that was generalized in [24] to a NET scenario.

B. Estimation in a nonextensive Tsallis setting

The phenomenal success of thermodynamics and statistical physics crucially depends upon certain mathematical relationships involving energy and entropy, that can be translated in an essentially intact form into Tsallis' generalized Thermostatistics formalism [15–19], whose main ingredient is the generalized entropy S_q, that in terms of the real parameter q reads

$$S_q \,=\, \langle\, \frac{1}{q-1}\, (1\,-\, f^{q-1})\, \rangle,\;\; q\,\epsilon\,\mathcal{R}, \tag{9}$$

and coincides with Shannon's S for $q = 1$. Indeed, much work has been recently devoted to i) show that many of these relationships are valid for *arbitrary* values of Tsallis' parameter q, and ii) to find appropriate generalizations for the rest. In this vein we just mention that, by suitably maximizing the entropy of the system, Curado and Tsallis [17] found that the whole mathematical (Legendre-transform based) structure of thermodynamics becomes *invariant* under a change of the $q-$value (from unity to any other real number), while the connection of NET both with quantum mechanics and with Information Theory was established in [18], where it was shown that all of the conventional Jaynes-Boltzmann-Gibbs [33] results suitably generalize to the Tsallis' environment. For more details see [15].

Of course, to verify that NET is useful, it is necessary to show that it appropriately describes certain physical systems with q-values that are different from unity. Much work in this respect has been performed recently (144 refereed papers at the time of this writing) . We may cite applications to astrophysical problems [19, 20], to Lévy flights [21], to turbulence phenomena [22], to simulated annealing [23], etc. The interested reader is referred to [15] for additional references.

A suitable generalization of (6) to a NET setting should involve replacing any probability distribution f involved in evaluating expectation values by Tsallis' generalized weights f^q. As shown in [24], one finds that Fisher's Information Measure in a Tsallis's nonextensive setting (generalized FIM I_q) reads

$$I_q = < \{\frac{\frac{df}{dx}}{f(x)}\}^2 >_q, \tag{10}$$

which can be abbreviate as GFIM. Instead of (7) we have to deal, of course, with

$$< \eta_{est}(y) >_q = \eta. \tag{11}$$

In [24] one uses (10) to

- generalize the Cramer-Rao bound [5].
- discuss connections between Tsallis' entropy, on the one hand, and Fisher's measure, on the other one.
- derive a suitably generalized form of the Frieden-Nikolov's bound [33] to the time derivative $\frac{dS_q}{dt}$. This alternative form connects the entropy increase to I_q.

C. The Frieden-Soffer EPI Principle

The Principle of Extreme Physical Information (EPI) is an overall physical theory that is able to unify several sub-disciplines of Physics [11]. In Ref. [11] Frieden and Soffer (FS) show that the Lagrangians in Physics arise out of a mathematical game between an intelligent observer and Nature (that FS personalize in the appealing figure of a "demon", reminiscent of the celebrated Maxwell's one). The game's payoff introduces the EPI variational principle, which determines simultaneously the Lagrangian *and* the physical ingredients of the concomitant scenario.

FS [11] envision the following situation: some physical phenomenon is being investigated so as to gather suitable, pertinent data. Measurements must be performed. Any measurement of physical parameters appropriate to the task at hand initiates a relay of information I (or I_q) from Nature (the demon) into the data. The observer acquires information, in this fashion, that is precisely I (or I_q). FS assume that this information is stored within the system (or inside the demon's mind). The demon's information is called, say, J (or J_q) [11].

Assume now that, due to the measuring process, the system is perturbed, which in turn induces a change δJ (δJ_q)of the demon's mind. It is natural to ask ourselves how the data information I will be affected. Enters here FS's EPI: *in its relay from the phenomenon to the data no loss of information should take place*. The ensuing new Conservation Law states that $\delta J = \delta I$, or, rephrasing it

$$\delta(I - J) = 0, \tag{12}$$

so that, defining an action $\mathcal{A}$

$$\mathcal{A} = I - J, \tag{13}$$

EPI asserts that the whole process described above extremalizes $\mathcal{A}$. FS [11] conclude that the Lagrangian for a given physical environment is not just an *ad-hoc* construct that yields a suitable differential equation. It possesses an intrinsic meaning. Its integral represents the physical information $\mathcal{A}$ for the physical scenario. On such a basis some of the most important equations of Physics can be derived [11]. For an interesting Quantum Mechanical derivation see [34].

Within the present context our demon is working in a non-extensive fashion, so that the EPI principle should read

$$\mathcal{A}_q = I_q - J_q, \tag{14}$$

i.e., using appropriate q-generalizations of the intervening physical quantities. Thus the Frieden-Soffer information transfer game is played here according to

$$\delta(I_q - J_q) = 0. \tag{15}$$

III. THE WORKINGS OF FS'S DEMON IN A NON-EXTENSIVE SCENARIO

A. Introductory remarks

One starts here the FS game by using (15), i.e., by extremalizing Fisher's generalized information for translation families

$$I_q = \int f(x)^{q-2} \dot{f}(x)^2 dx, \tag{16}$$

and considering a physical scenario in which the knowledge of the information demon [11] is limited to that concerning the normalization constraint. The demon's J_q is

$$J_q = \int \gamma_0 f(x) dx, \tag{17}$$

where γ_0 is the pertinent Lagrange multiplier. The demon's information is reduced here to its minimum-core expression.

Playing the Frieden-Soffer game then leads to

$$-2 \ddot{f}(x) + (2-q) \frac{\dot{f}(x)^2}{f(x)} - \gamma_0 f(x)^{2-q} = 0, \tag{18}$$

q-dependent, non-linear differential equation that should yield our "optimal" probability distribution f. It easy to show that (18) has the first integral

$$f^{q-2}\dot{f}^2 + \gamma_0 f = c \tag{19}$$

and

$$x - x_0 = \pm \int \frac{f^{\frac{q}{2}-1}}{\sqrt{c - \gamma_0 f}} df \tag{20}$$

is the final expression for the general solution of (18). Of course, c and x_0 are arbitrary integration constants.

B. The $q \neq 1$ environment

¿From (20) we see that $c > \gamma_0 f$ in order to obtain a real probability distribution f. This condition holds in several circumstances, as follows:

- For $c > 0$ and $\gamma_0 > 0$, the probability distribution $f < c/\gamma_0$, i.e., it has an upper positive limit and it is bounded.
- For $c > 0$ and $\gamma_0 < 0$, the probability distribution $f > c/\gamma_0$, and thus it has a negative lower limit and it is unbounded.
- For $c < 0$ and $\gamma_0 > 0$, the probability distribution $f < c/\gamma_0$, i.e., it is negatively defined and we must reject it.
- For $c < 0$ and $\gamma_0 < 0$, the probability distribution $f > c/\gamma_0$, i.e., it has a positive lower limit and it is unbounded.

We conclude that the first possibility is the only one that has a physical meaning. Indeed, inserting $f(x_m) = c/\gamma_0$ into (19), we obtain $\dot{f} = 0$. Thus, at x_m the probability distribution f has an extremum. On the other hand, inserting these results into (18) we have $\ddot{f} = -\frac{\gamma_0}{2}\left[\frac{\gamma_0}{c}\right]^{q-2}$ (negatively defined). An f-maximum at x_m ensues.

The particular case $c = 0$ is consistent with $\gamma_0 < 0$ and needs separate analysis. Here, the momoparametric solution of (20) is given by

$$f = \left[\pm\frac{(q-1)\sqrt{-\gamma_0}}{2}(x - x_0)\right]^{\frac{2}{q-1}}. \tag{21}$$

This expression would yield a probability distribution that cannot be normalized if x ranges from minus to plus infinity.

C. Relativistic scenario

Evaluating derivatives (with respect x) in (18), and changing variables to

$$u = \frac{\dot{f}(x)}{f(x)}\ , \tag{22}$$

we obtain

$$\ddot{u}\ + \alpha\ u\ \dot{u}\ + \beta\ u^3 = 0\ , \tag{23}$$

with

$$\alpha\ =\ (2\ q - 1)\ , \tag{24}$$

and

$$\beta = -\frac{1}{2}\ q\ (1 - q)\ . \tag{25}$$

Equation (23) arises in several interesting physical problems as, for instance, in the case of i) Einstein's field equations for homogeneous, isotropic and spatially flat cosmological models with no cosmological constant [29, 35–40], and ii) Bianchi I-type metrics [41], for viscous fluids and a selfinteracting exponential potential scalar field.

We shall tackle (23) by recourse to ideas presented by Chimento and Jakubi [35], who show that its more general solution that can be *explicitly* writen as a function of x obtains for the special value $\alpha^2/\beta = 9$. In our case, this assertion translates itself into two possible q-values, namely:

- $q = 2$, and
- $q = -1$,

as will be discussed below.

Following [35], we perform the nonlocal change of variables

$$z\ =\ \frac{1}{2}\ u^2, \tag{26}$$

and obtain

$$\frac{d^2 z}{d\eta^2}\ +\ \frac{dz}{d\eta}\ +\ \beta^{'}\ z =\ 0, \tag{27}$$

which is a linear ordinary differential equation, with

$$d\eta\ =\ \alpha\ u\ dx, \tag{28}$$

and

$$\beta^{'}\ =\ 2\ \frac{\beta}{\alpha^2}. \tag{29}$$

The general solution of (27) is of the form

$$z(\eta)\ =\ A_1\ \mathrm{e}^{\lambda^{+}\ \eta}\ +\ A_2\ \mathrm{e}^{\lambda^{-}\ \eta}, \tag{30}$$

where

$$\lambda^{\pm} = -\frac{1}{2} \pm \frac{1}{2|\alpha|} \tag{31}$$

are the roots of the pertinent characteristic equation, A_1, A_2 being integration constants.

Going all the way back from z to f we obtain the most general distribution function that verifies the Frieden-Soffer tenets. However, as far as one knows at the present stage [35], the road back can be traversed in *explicit, analytical* fashion only for

$$\frac{\beta}{\alpha^2}\ =\ \frac{1}{9}, \tag{32}$$

which underlines the importance of the above mentioned special q-values (2 and -1) . In the next section we stress the connection between the probability distribution and the cosmological expansion factor, identifying both physical quantities via equations (23) and (27).

IV. COSMOLOGICAL APPLICATIONS

Due to their nonlinear nature, exact solutions to Einstein's equations cannot easily be obtained. However, one finds diverse problems of physical interest where Einstein field equations (EFE) can be cast as particular instances of a second order nonlinear differential equation of the type [2]

$$\ddot{y} + \alpha g(y)\dot{y} + \beta g(y)\int dy g(y) + \gamma g(y) = 0, \tag{33}$$

where $y = y(x)$ and $g(y)$ is a real function. α, β, γ are constant parameters.

As examples of the preceding assertion, we may mention the case of EFE i) for homogeneous, isotropic and spatially flat cosmological models with no cosmological constant [35–40], ii) for a time decaying cosmological constant [42], and iii) for Bianchi I-type metrics with a variety of matter sources [41].

A. The expansion rate of the Universe: different scenarios

It is thought that quantum effects played a leading role in the early Universe, as, for instance, particle production and vacuum polarization, that arise within a quantal framework. It is known that these two phenomena can be modeled in terms of a classical bulk viscosity [43]. By recourse to the so-called Extended Irreversible Thermodynamics [44, 45], a relativistic, second-order off-equilibrium approach, one can deal with a spatially flat homogeneous and isotropic space-time, filled with a causal viscous fluid and described by the Friedmann-Robertson-Walker line element [46]

$$ds^2 = dt^2 - a^2(t)\left[dx_1^2 + dx_2^2 + dx_3^2\right], \tag{34}$$

with the full version of the transport equation for the bulk viscous pressure σ

$$\sigma + \tau\dot{\sigma} = -3\zeta H - \frac{1}{2}\epsilon\tau\sigma\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right). \tag{35}$$

where $a(t)$ is the scale factor, $H = \dot{a}/a$ is the expansion rate, τ is the characteristic timescale for linear relaxational effects, ζ is the linear bulk viscosity and T is the equilibrium temperature.

In [35] it was considered a causal viscous fluid whose equilibrium pressure obeys a γ-law equation of state, while the transport equation for the viscous pressure σ was taken with $\epsilon = 0$, which corresponds to the so-called truncated theory. There, it was shown that H satisfies a modified Painlevé-Ince equation that, in turn, can be cast into the form of (33), i.e.,

$$\ddot{H} + \alpha H\dot{H} + \beta H^3 = 0 \tag{36}$$

with $f(y) = y$ and $\gamma = 0$ [2]. Comparing the latter with (23) we get $u = H$, and, a fortiori $a(t) = f(x)$ (from (22)) . This result shows that *the probability distribution can be identified with the expansion parameter for the evolution of the universe by choosing the time variable t proportional to the information parameter x*. This suggest that the solutions of (18) representing probability distributions without physical meaning, becuase they cannot be normalized, may be associated with the expanding (FRW) cosmological model. So that, we have a more extended background where the (FS)'s demon along with non-extensive scenario can be used.

Cosmological models with a viscous fluid source have been also studied using the *full* causal Irreversible Thermodynamics [29, 39, 40, 47]. Relating the temperature T to the energy density in the simplest possible fashion that is able to guarantee a positive heat capacity, one finds that the expansion rate H verifies

$$H\ddot{H} - (1 + r)\,\dot{H}^2 + AH^2\dot{H} - BH^4 = 0 \tag{37}$$

where $r > 0$, while A, B and γ_0 are constants. By recourse to an adequate choice of variables ($H = y^n$, with $n = -1/r$) in Eq.(37), the above equation becomes (again) identical to (33) [2, 29], i.e., we can write

$$\ddot{y} + \alpha y^n\dot{y} + \beta y^{2n+1} = 0 \tag{38}$$

with $f(y) = y^n$ and $\gamma = 0$.

For a still different scenario consider now the early time evolution of a dissipative universe, in the relaxation dominated regime. One is again lead here to an equation for the expansion rate H that can be identified with (23) and (33) [48]. Making once more the identification $u = H$, Eq.(38) turns into Eq. (23), so that $a(t) = f(x)$, and the probability distribution can, as before, be identified with the expansion factor in the same manner as we did above.

Let us discuss now the case of a perfect fluid source, satisfying an equation of state $p = (\sigma - 1)\rho$ with cosmological constant. The 00 Einstein equation

$$H^2 = \frac{1}{3}\frac{\rho_0}{a^{3\sigma}} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{39}$$

transforms itself into Eq.(33), with $f(y) = y$, and

$$\ddot{H} + (2+3\sigma)H\dot{H} + 3\sigma H^3 - \sigma\Lambda H = 0 \tag{40}$$

provided we derive twice Eq.(17). Thus, $a(t) = f(x)$ in a universe with no cosmological constant.

Another interesting illustration of Eq. (33) can be obtained by considering an anisotropic Universe, described by a Bianchi type I metric,

$$ds^2 = e^{f(t)}\left(-dt^2 + dz^2\right) + G(t)\left(e^{p(t)}\,dx^2 + e^{-p(t)}\,dy^2\right) \tag{41}$$

that is driven by a minimally coupled scalar field with an exponential potential [2]. The Klein-Gordon equation for the scalar field and the Einstein equations for the metrics are expressed in terms of the semiconformal factor G and of their derivatives, as follows [2]

$$\dot{\phi} = \frac{m}{G} - \frac{k}{2}\frac{\dot{G}}{G}, \tag{42}$$

$$\dot{p} = \frac{a}{G}, \tag{43}$$

$$e^f = \frac{\ddot{G}}{2GV}, \tag{44}$$

$$\frac{\ddot{G}}{G} - \frac{1}{2}\left(\frac{\dot{G}}{G}\right)^2 - \frac{\dot{G}}{G}\dot{f} + \frac{1}{2}\dot{p}^2 = -\dot{\phi}^2, \tag{45}$$

where m and a are arbitrary integration constants. The solutions for this set of equations can be obtained if one is able to solve the following equation for G

$$G\frac{\ddot{G}}{\dot{G}} + (K-1)\dot{G} + \frac{M^2}{\dot{G}} = c, \tag{46}$$

where K is a constant parameter and M, c are integration constants. Once $G(t)$ is known, one can compute the field $\phi(t)$ and the remaining metric functions $p(t)$, $f(t)$ from Eq.(42), Eq.(43), and Eq.(44), respectively. Making

$$G = y^{1/K}, \qquad \textit{and} \qquad \tau \equiv -ct, \tag{47}$$

Eq.(46) becomes an equation of the type Eq.(33)

$$y'' + y^{-n}y' + \frac{M^2}{nc_3^2}y^{1-2n} = 0, \tag{48}$$

where a prime denotes the derivative with respect to τ and $n = 1/K$ [41].

B. Time variable and information variable

It should become clear by now that it is of great interest to investigate Eq.(33) from *all conceivable points of view.* Using nonlocal transformations this equation can be first linearized and then solved with the only proviso of the form invariance of Eq.(33) for an arbitrary function $f(y)$.

Now, if we relate y to the probability distribution function f of the former Sections in the fashion

$$y = \frac{\dot{f}}{f}, \tag{49}$$

and set

$$H(t) = y, \tag{50}$$

together with the choice: *time variable t proportional to the information parameter x*, one immediately realizes that equation(33) translates itself into equation (23), the protagonist of our discussion in Section III. This shows that the probability distribution, in many circumstances, can be identified with the scale factor $a(t)$ of the universe. In such a case, taking into account that the universe has a final, matter dominated Friedmann stage, the physical range of the Lagrange multiplier is given by $\gamma_0 < 0$. For $c > 0$ the solutions stem from a singularity. However, for $c < 0$ the solutions avoid the initial singularity. In the particular case $c = 0$ the solutions (21) represent a universe beginning or ending in a singularity at $x = x_0$, for $q > 1$. For $q < 1$ the solutions diverge at the finite proper time $x = x_0$.

The special mapping: (information $\rightarrow$ time) that arises in natural fashion from the present considerations allows one to give a degree of plasibility to the particular Frieden-Soffer solutions with $c = 0$ (21), since the scale factor, and its associated probability density, represent the evolution of universe. In such circumstances, the scale factor is, of course, unbounded, it does not make any sense to try normalizing it.

Summing up: we are leading to a variety of cosmological problems involving Einstein's field equations which via a nomlocal transformation and the special mapping: (information $\rightarrow$ time) *can be reinterpreted* as a problem within a non-extensive scenario, by application of the nonextensive EPI Principle.

V. CONCLUSIONS

In the present communication we have achieved the following results

- We translated the rules of the Frieden-Soffer Information Transfer game into a NET parlance.
- By suitably playing the game, we found exact analytical solutions to special instances of Einstein's field equations.

- Nonlocal variable transformations of differential equations have allowed for a widening of the class of differential equations (particularly of the second order) which can be linearized and solved.

In essence our procedure provides a method for determining a useful nonlocal symmetry for a set of differential equations characteristic of several different physical problems. However, these problem can be seen to be are equivalent under nonlocal variable transformations. *In that peculiar sense*, we see that some cosmological solutions can be regarded as probability distributions, pertaining to a $q \neq 1$ NET environment, that maximize Fisher's information measure. One may think that Frieden-Soffer's demon is able to map the space-time continuum into a probability space. The solutions here studied can then be added to the impressive collection elaborated by Frieden and Soffer [11].

That a NET environment may be the appropriate one in order to address physical problems connected with gravitation can be understood on the grounds that two systems A and B that interact in such a fashion can never be really "separated" (and become isolated ones) as additivity would demand, since they will always "feel" each other's presence on account of the infinite range of the associated force. Thus

$$S_{(A+B)} \neq S_A + S_B \tag{51}$$

i.e., a non-extensive treatment becomes mandatory. Indeed, a classical problem related to Galactic models that had defied solution within an extensive Shannonian context has been successfully solved appealing to Tsallis' NET [19]. The present results should be viewed within such a background-landscape.

As a last comment of this paper we mention that the results concerning to the equivalence of some expanding (FRW) cosmological model and the problem related with the (FS)'s demon in a non-extensive scenario are not consequence of the equivalence principle. They are a diret consequence of the nonlocal symmetries that have certain nonlinear differential equations, in our case, the Einstein's field equations for a spatially flat (FRW) universe and the nonlinear differential equation (18), resulting for a simple variational calculus of the Fisher's generalized information for translation families. This nonlocal symmetries can be realized by means of nonlocal variable transformation. They mapped different physical problem changing the number of point symmetry generators and the very physics of the original problem. However, the final motion equations are the same giving rise to a larger class of phisical problem which can be considered "equivalent" under a most general nonlocal symmetry group. In a forthcomming paper we shall investigate the connection between the inertia of accelerative effects and the statistics.

Acknowledgments

The authors are indebted to the PROTEM Program of the National Research Council (CONICET) of Argentina for financial support. The assistance of C. Mostaccio is gratefully acknowledged. One of us (L.P.C) is indebted to the University of Buenos Aires for financial support under Grant EX-260.

[1] *Differential Equations* (Cambridge University Press, Cambridge, 1989)
[2] L. P. Chimento, *J. Math. Phys.* **38**, (1997) 2565.
[3] R. A. Fisher, *Statistical Methods and Scientific Inference* , second edition (Oliver and Boyd, London, 1959).
[4] R. A. Fisher, *Proc. Camb. Soc.* **22**, 700 (1925).
[5] B. R. Frieden, *Phys. Lett. A* **169**, 123 (1992).
[6] B. R. Frieden in *Advances in Imaging and Electron Physics* , edited by P. W. Hawkes (Vol. 90, pp. 123-204, Academic, Orlando, 1994).
[7] B. R. Frieden, *Physica A* **198**, 262 (1993).
[8] B. R. Frieden and R. J. Hughes *Phys. Rev. E* **49**, 2644 (1994).
[9] B. Nikolov and B. R. Frieden , *Phys. Rev. E* **49**, 4815 (1994).
[10] B. R. Frieden, *Phys. Rev. A* **41**, 4265 (1990).
[11] B. R. Frieden and B. H. Soffer, *Phys. Rev. E* **52**, 2274 (1995).
[12] B. R. Frieden, *Am. J. Phys.* **57**, 1004 (1989).
[13] D. Brody and B. Meister, *Phys. Lett. A* **204**, 93 (1995).
[14] A. R. Plastino and A. Plastino, *Phys. Rev. E* **52** (1995).
[15] C. Tsallis, Fractals **6**, 539 (1995), and references therein.
[16] C. Tsallis, *J. Stat. Phys.* **52**, 479 (1988).
[17] E.M.F. Curado and C. Tsallis, *J. Phys. A* **24**, L69 (1991) ; Corrigenda: **24**, 3187 (1991) and **25**, 1019 (1992).
[18] A. R. Plastino and A. Plastino, *Phys. Lett. A* **177**, 177 (1993).
[19] A. R. Plastino and A. Plastino, *Phys. Lett. A* **174**, 384 (1993).
[20] A. R. Plastino and A. Plastino, *Phys. Lett. A* **193**, 251 (1994).
[21] P. A. Alemany and D. H. Zanette, *Phys. Rev. E* **49**, 956 (1994).
[22] B. M. R. Boghosian, *Phys. Rev. E* **53**, 4754 (1995).
[23] T. J. P. Penna, *Phys. Rev. E* **51**, R1 (1995).
[24] A. Plastino, A. R. Plastino and H. G. Miller, *Physica A* **235**, (1997) 577.
[25] F. Pennini and A. Plastino. *Physica A* **246** (1997) in press.
[26] L. P. Chimento, *Class. Quantum Grav.* **6**, (1989) 1285.
[27] L. P. Chimento, *Gen. Rel. Grav.* **25**, (1993) 979.
[28] M. G. Alé and L. P. Chimento, *Class. Quantum Grav.* **12**, (1995) 101.
[29] L. P. Chimento and A. S. Jakubi *Class. Quantum Grav.* **14** (1997) 1811.
[30] S. Weinberg, *Gravitation and Cosmology. Principles and Applications of the General Theory of Relativity* (Wiley, 1972).

[31] J. M. Corcuera, *Catalonian Math. Soc. Bull.*, bf 11 (1996) 47 ; J. M. Oller, in *Statistical data analysis and inference*, Elsevier, Amsterdam, 1989, pp 41-58; J. M. Oller and J. M. Corcuera, *Annals of Statistics* (1995) 1562.

[32] R. N. Silver, in *Essays in honor of Edwin T. Jaynes*, ed. by W. T. Grandy, Jr. and P. W. Milonni, Cambridge University press, Cambridge (1993).

[33] E. T. Jaynes in *Statistical Physics*, ed. W. K. Ford (Benjamin, NY, 1963); A. Katz, *Statistical Mechanics*, (Freeman, San Francisco, 1967).

[34] M. Casas, F. Pennini and A. Plastino. *Physics Letters A* (1997) in press.

[35] L. P. Chimento and A. S. Jakubi, *Class. Quantum Grav.* **10**, 2047 (1993).

[36] L. P. Chimento, *Proceedings of the First Mexican School on Gravitation and Mathematical Physics* (Guanajato, Mexico, 1994).

[37] L. P. Chimento and A. S. Jakubi, *Proceedings of the First Mexican School on Gravitation and Mathematical Physics* (Guanajato, Mexico, 1994).

[38] L. P. Chimento and A. S. Jakubi, *Phys. Lett. A* **212** , 320 (1996).

[39] L. P. Chimento, A. S. Jakubi and V. Méndez *Inter...* (in press).

[40] L. P. Chimento, A. S. Jakubi, V. Méndez and R. Maartens *Class. Quantum Grav.* (in press).

[41] J. M. Aguirregabiria and L. P. Chimento, *Class. Quantum Grav.* (in Press).

[42] M. Reuter and C. Wetterich, Phys. Lett. B **188**, 38 (1987).

[43] B. L. Hu, *Phys. Lett. A* **90**, 375 (1982).

[44] D. Pavon and J. Casas-Vazquez, *Ann. Inst. H. Poicare* A **36**, 79 (1982).

[45] D. Jou, J. Casas-Vazquez, and G. Lebon, *Extended irreversible thermodynamics* (Springer, Berlin, 1993).

[46] D. Pavon, J. Bafaluy, and D. Jou, *Class. Quantum Grav.* **8**, 347 (1991).

[47] V. Mendez and D. Jou, *Qualitative analysis of causal cosmological models* (Preprint, Universidad Autonoma de Barcelona, 1996).

[48] M. Zakari and D. Jou, *Phys. Lett.* A **175**, 395 (1993).